

# Valuing science policy: Dynamic decisionmaking with generalized Bayesian learning

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TWEEDS



# Also known as..

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# How to approximate and integrate high dimensional objects quickly and accurately

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**Current expenditures:** \$2-3 billion/year, almost 50% of US climate change expenditures (GAO 2018)

**What is the marginal benefit of funding climate science?**



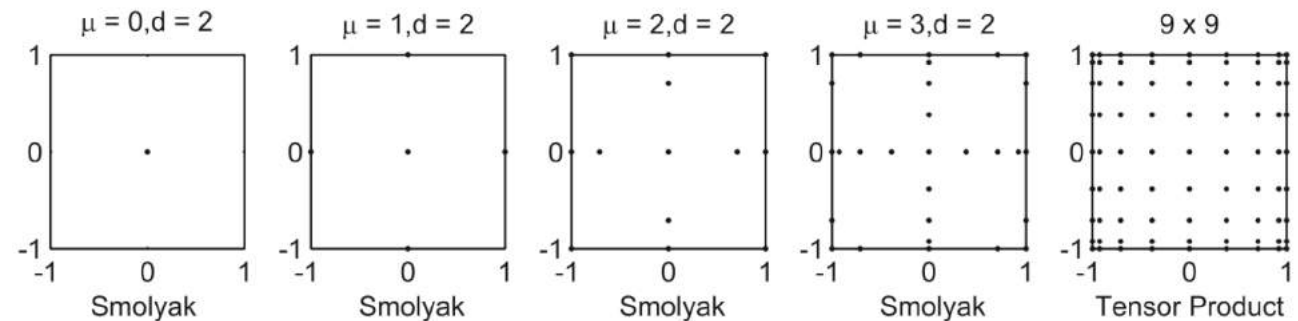
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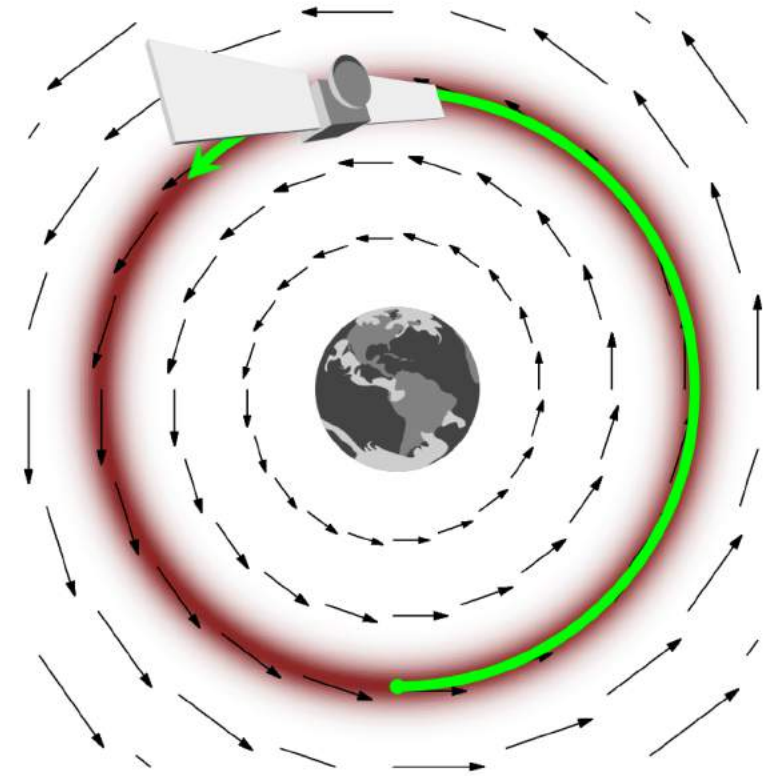


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Sparse grids

Hamiltonian Monte Carlo



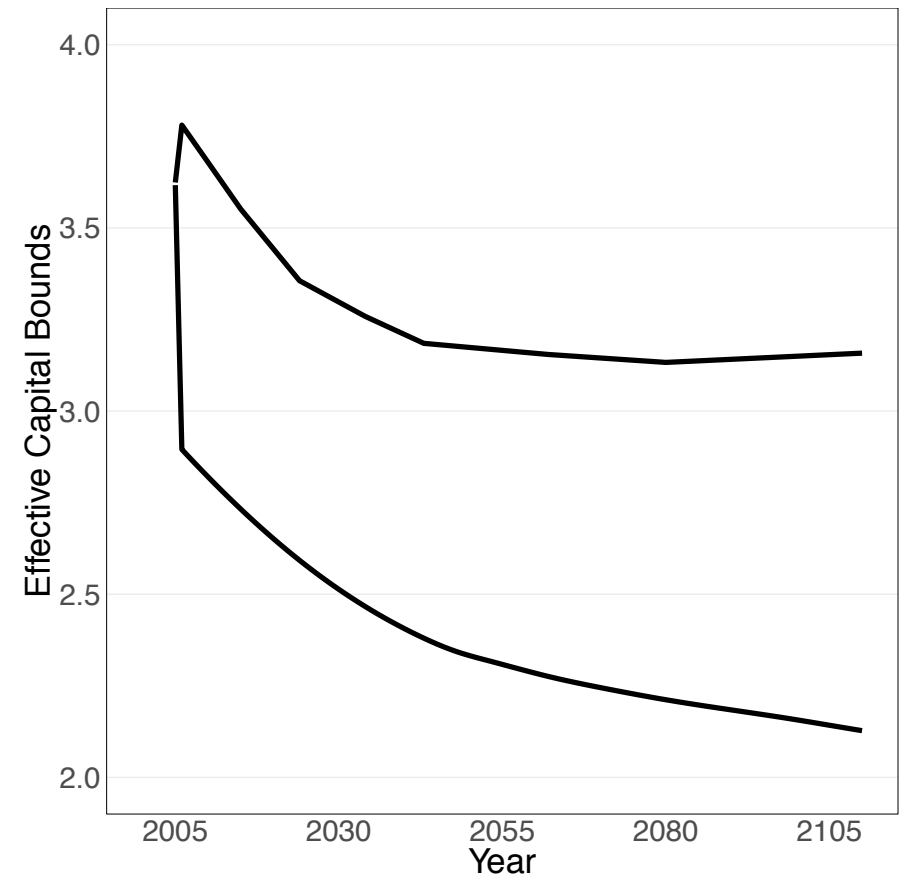
# We estimate the value of science policy that helps us learn about climate sensitivity

(sort of) new computational methods we use:

Sparse grids

Hamiltonian Monte Carlo

Adaptive grids / stochastic simulation



# Why do we need these methods?

## Sparse grids

- 200 states

## Hamiltonian Monte Carlo

- Nonconjugate distributions
- High dimensional Bayesian estimation

## Adaptive grids / stochastic simulation

- 200 states



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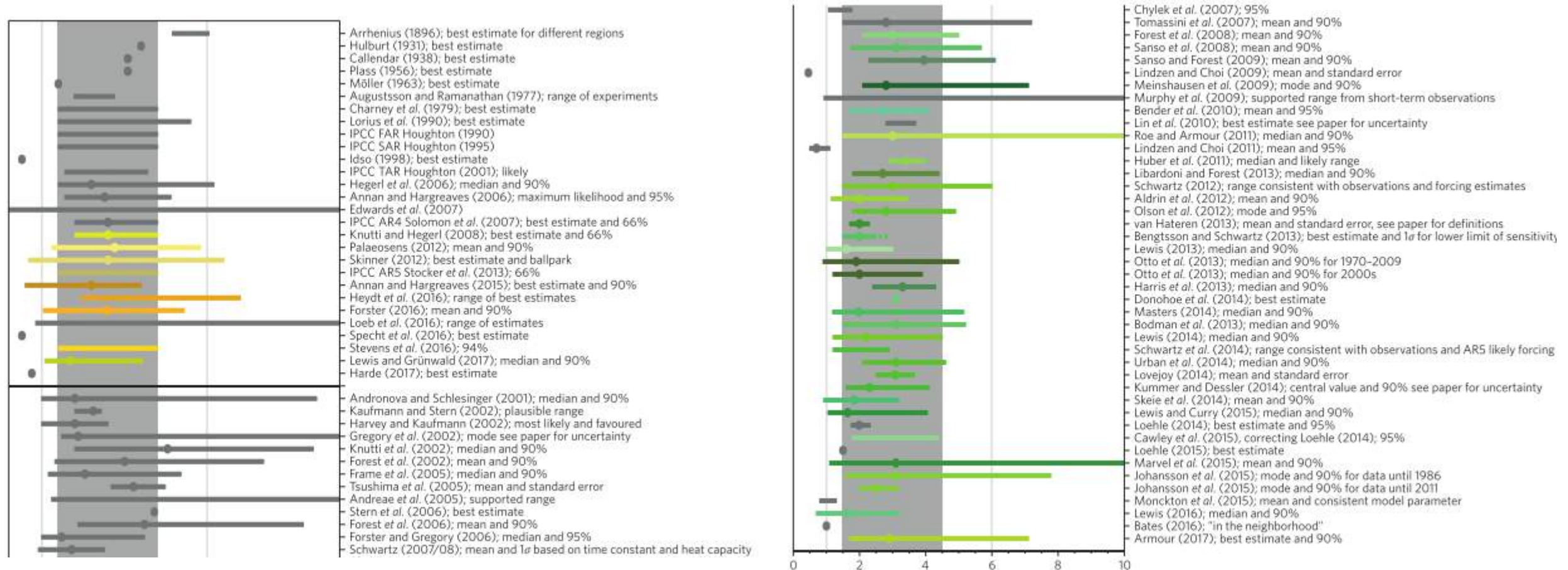
1 million core-hours

Optimistic human time equivalent:

1 month on UA Ocelote (~1,400 cores)



# Climate sensitivity: equilibrium warming from a doubling of CO<sub>2</sub>



“The estimated range of the ECS has not changed much despite massive research efforts.” - Knutti et al. (2017)

# What is the value of science policy?

1) Science is **extremely** valuable

Accelerating learning is worth up to:

- \$100s of billions annually
- \$1000s per capita lump sum today
- 1% permanent consumption gain



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2) Science should be funded at **large scale**

There are increasing returns to scientific information

# IAM + generalized Bayesian estimation

The model is:

- Dynamic stochastic DICE with storage of observed histories
- Bayesian estimation routine for the temperature transition

$$T_{t+10} = T_t + C1 \left[ F(M_{t+10}, t + 10) - \frac{f}{CS} T_t - C3(T_t - O_t) \right] + v_{t+10}^T$$

We want the **CS** distribution (state of climate knowledge)

The equation doesn't admit a closed form Markov updating rule → we need to store observed histories, estimate generic posteriors

# How we do it: 5 steps

- 1) Collocation
- 2) Interpolation
- 3) Learning
- 4) Belief computation
- 5) Dynamic programming

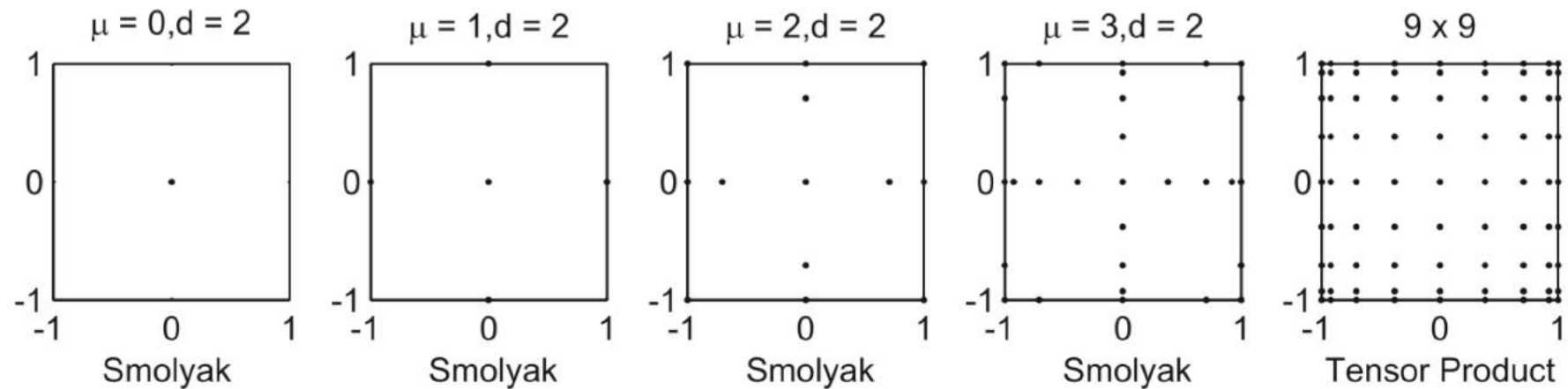


**Step 1: Make an approximation grid**

**Collocation**

# Taking nonconjugate learning to a climate-economy model: collocation sparsity

We use a **sparse collocation grid** (Smolyak, 1963)



**Intuition:** Optimal scheme to minimize approximation errors with a given number of grid points

**Delivers:** polynomial complexity instead of exponential

**Why:** we need to store observed histories over centuries

**Step 2: Generate data**

**Interpolation**

# Issue: model is way too big on a 1 year timestep

We use a 10 year time step for policymaking because of computational limitations (hundreds vs thousands of states)

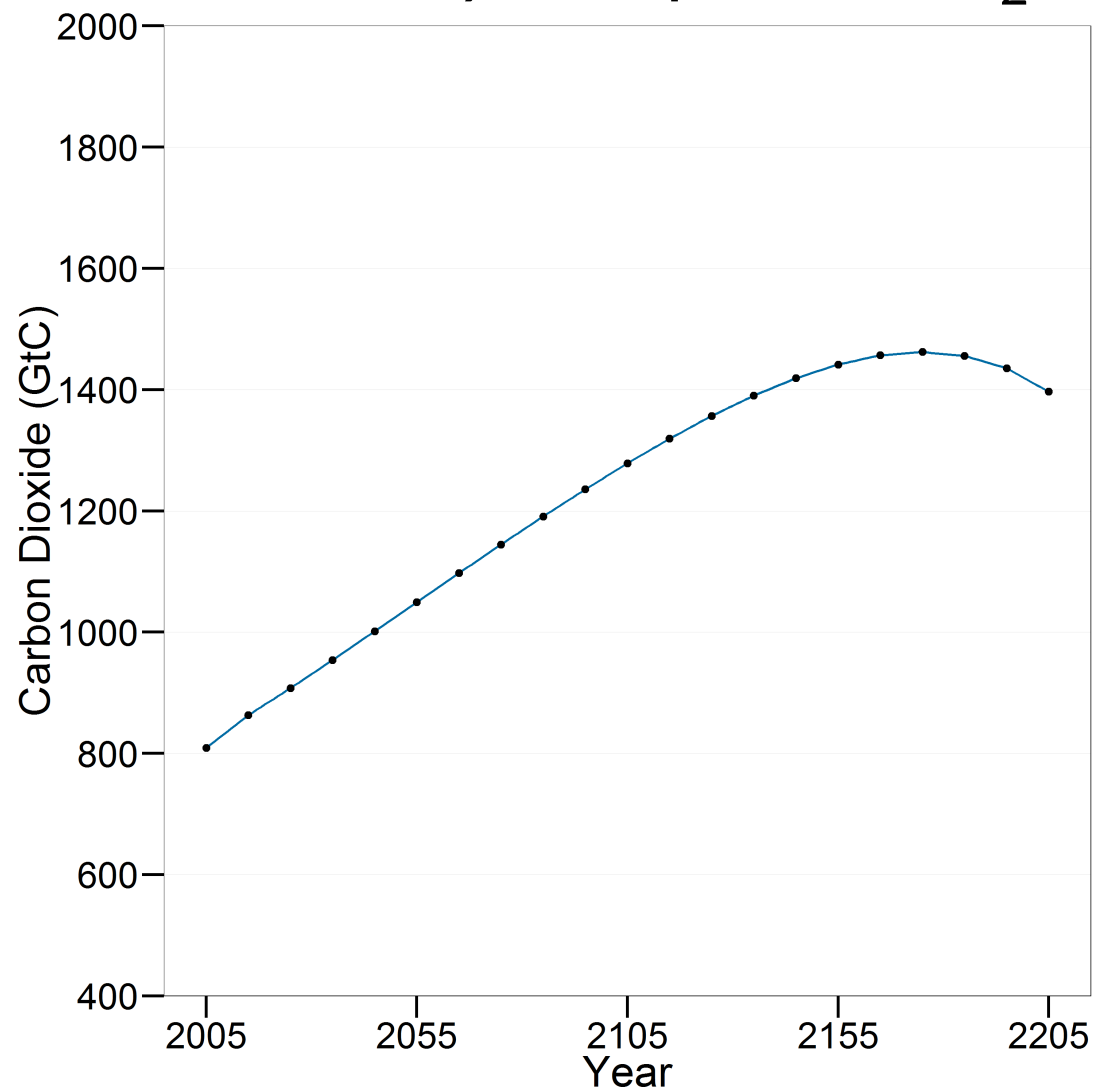
This is obviously bad for learning about climate sensitivity

**Solution:** interpolate between 10 year histories using Brownian bridges + annual climate dynamics

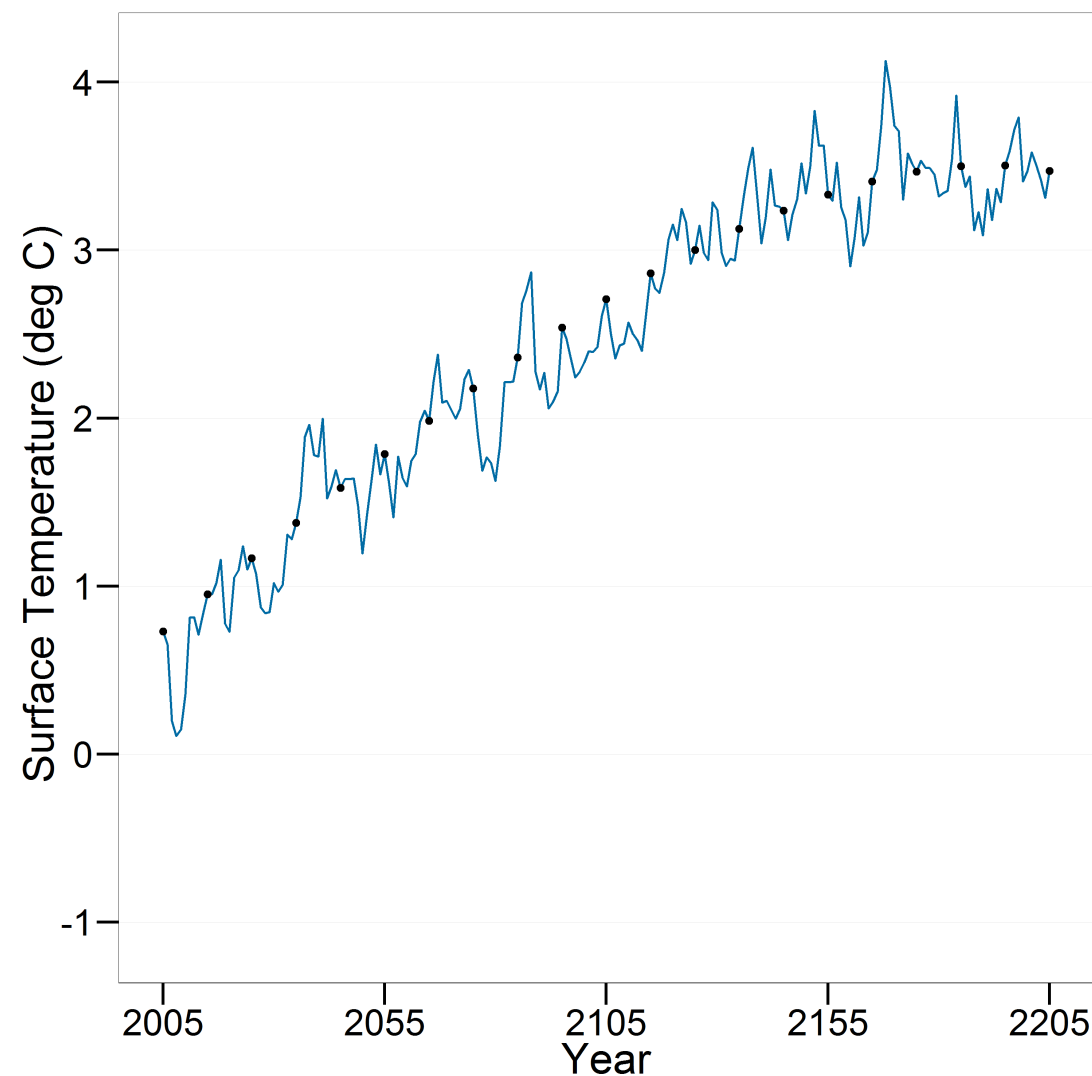


# Our interpolated history

## Linearly interpolate CO<sub>2</sub>



## Bridge + annualized transition



# Step 3: Bayes' rule

Learning

# Step 3: Bayes' rule



# Step 4: Hamiltonian Monte Carlo

Beliefs

# We need a usable posterior for our model

Bayes gives us a posterior, now we need a way to approximate it to use it in the model

**Hamiltonian Monte Carlo:** new and efficient method for sampling high dimensional distributions

We need HMC vs MCMC because we will be estimating a  $\sim 70$  dimensional posterior (CS + data + volatility) to calibrate our model (e.g. Aldrin et al. 2012; Skeie et al. 2014)



# Why HMC is the coolest

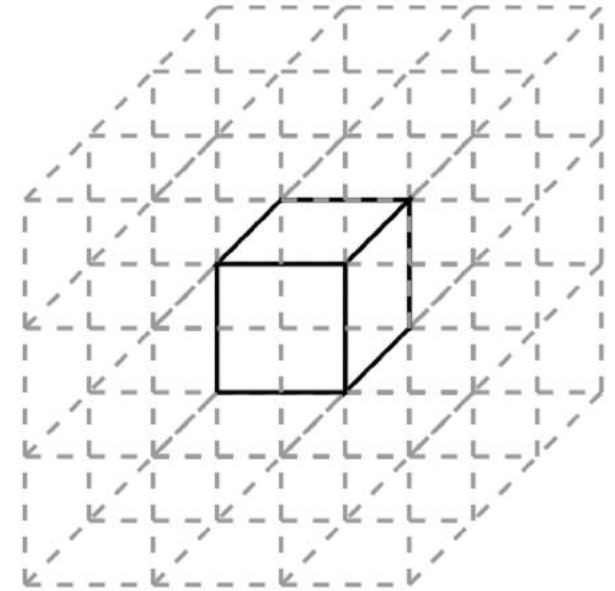
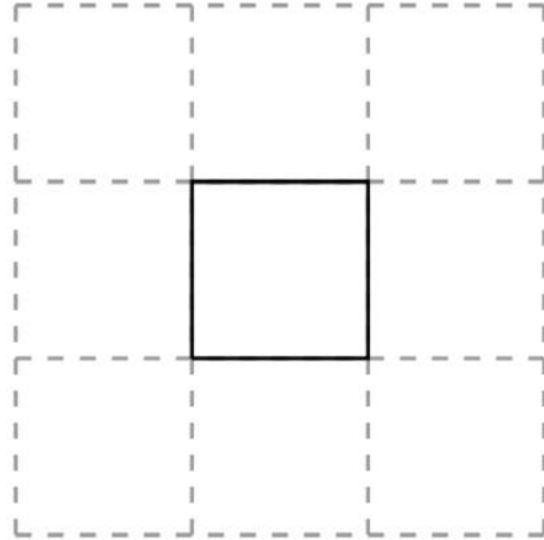
We commonly take expectations by exploring the distribution through random walks (e.g. Metropolis-Hastings)

High dimensional spaces pose problems for random walks

Don't efficiently traverse the important parts of the distribution

Why? See Betancourt (2017) for details and pictures

# Volume scales exponentially in # of dimensions



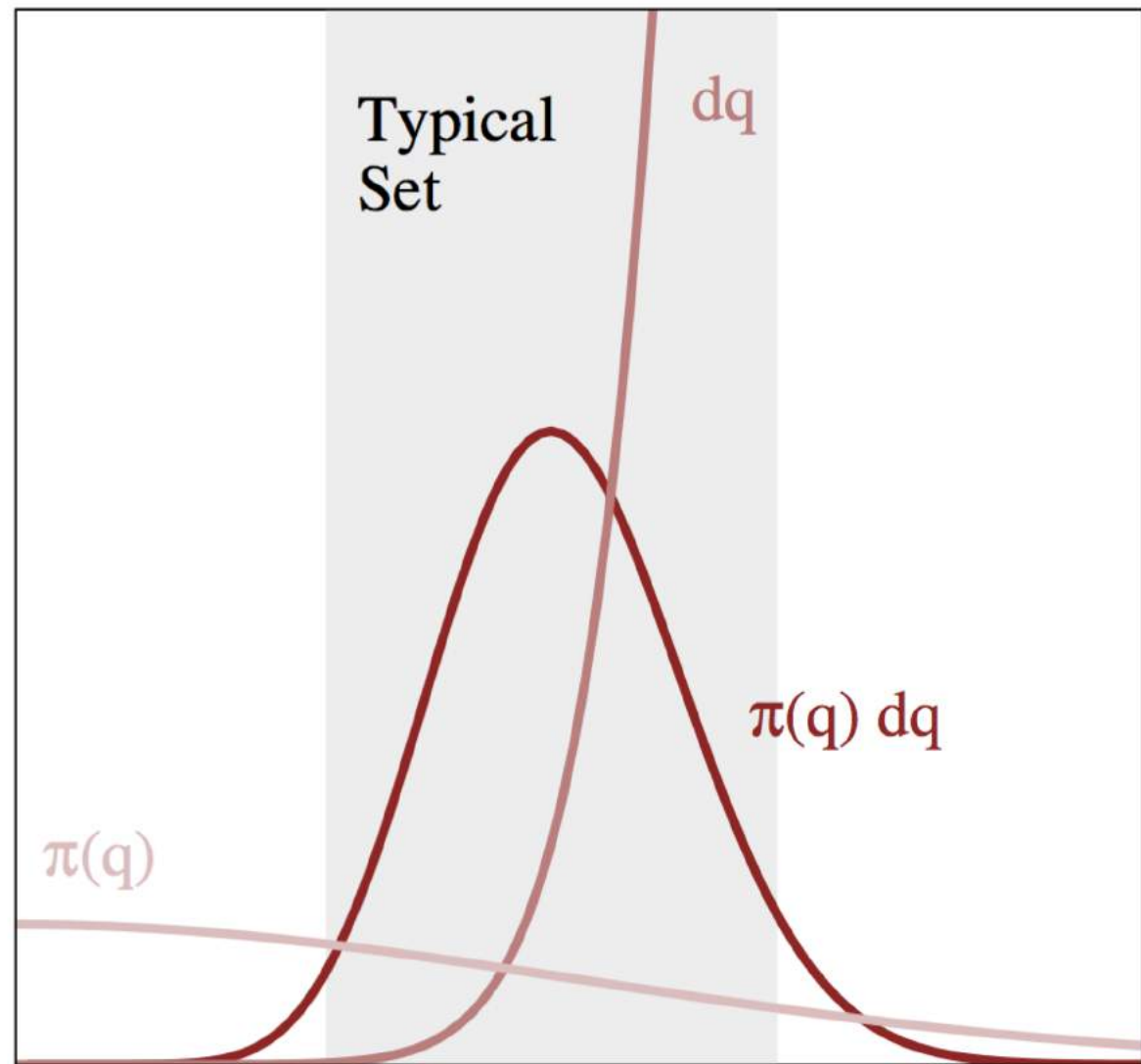
High density regions (e.g. the distributional mode) take up smaller and smaller volumes as the dimensionality increases

# Density vs volume tension in high dimensions

A density concentrates around its mode but the vast majority of volume is away from the mode

Contributions to the expectation are determined by the product of density and volume

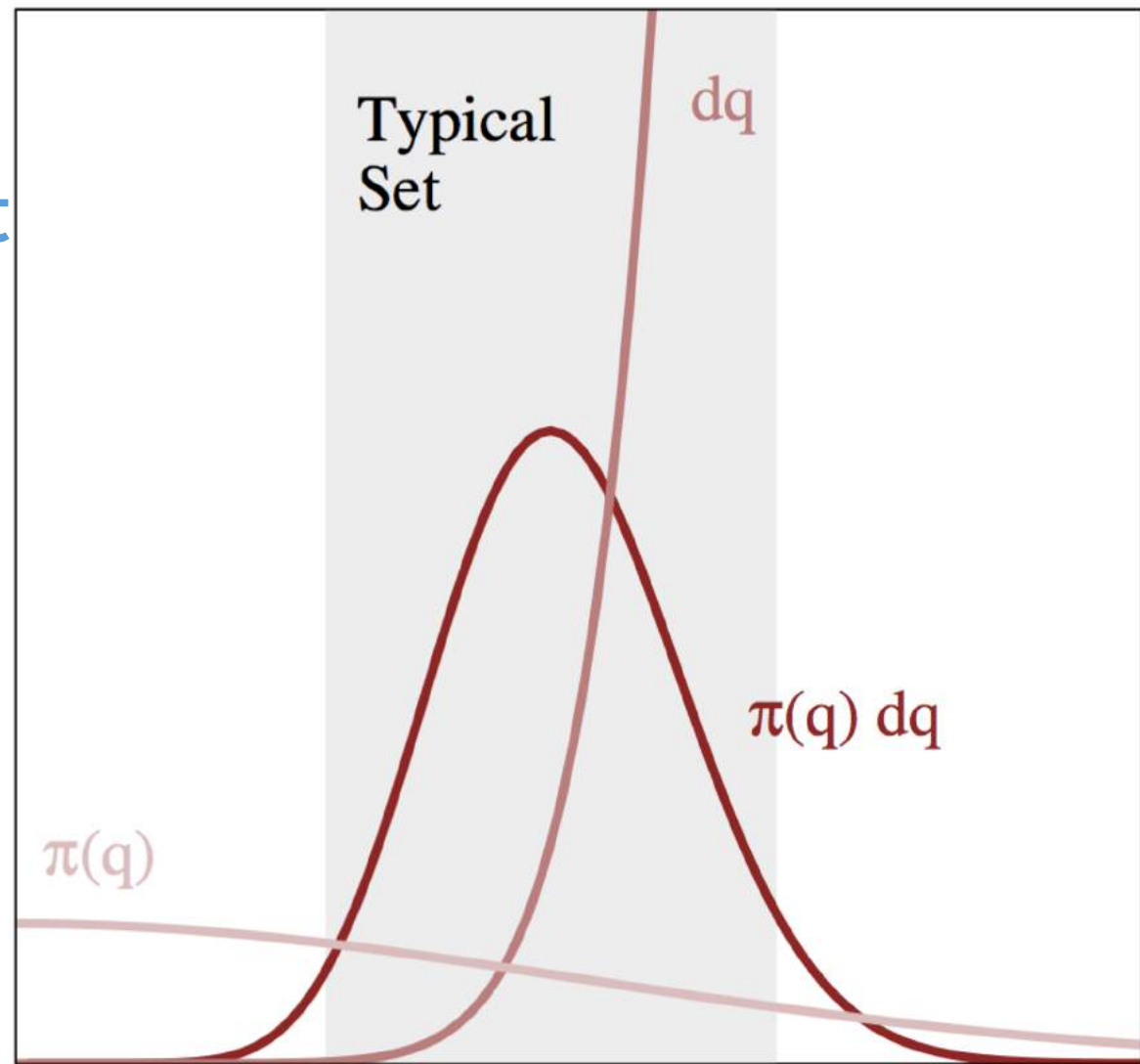
We don't need (or want) to explore the entire distribution



# Density vs volume tension in high dimensions

The contributions are centered in an (small) area called the **typical set**

To efficiently take expectations we need to identify and focus on the typical set



# It's just physics

HMC takes standard MCMC approaches,  
but informs transitions so they closely follow the typical set

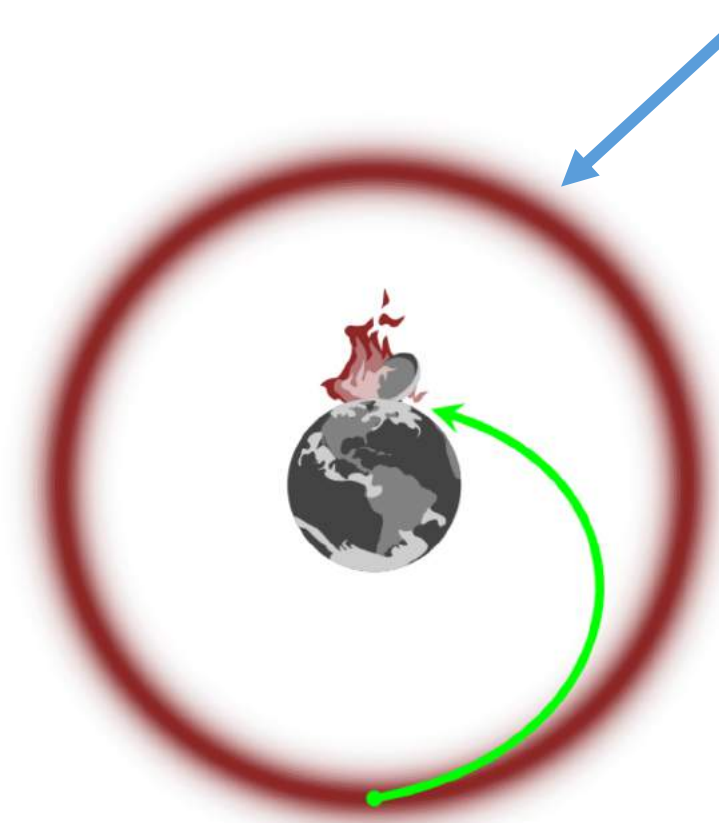
How? Ideas from classical physics

The typical set is actually very similar to stable orbits →

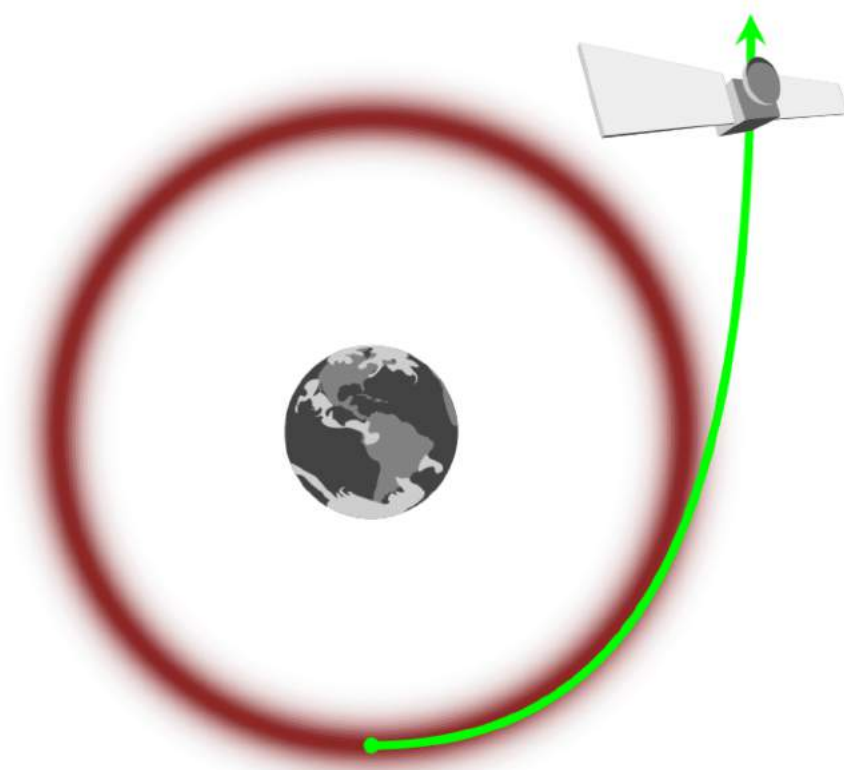
We need enough momentum to offset gravity (gradient's)  
pull toward Earth (the mode)

# It's just physics

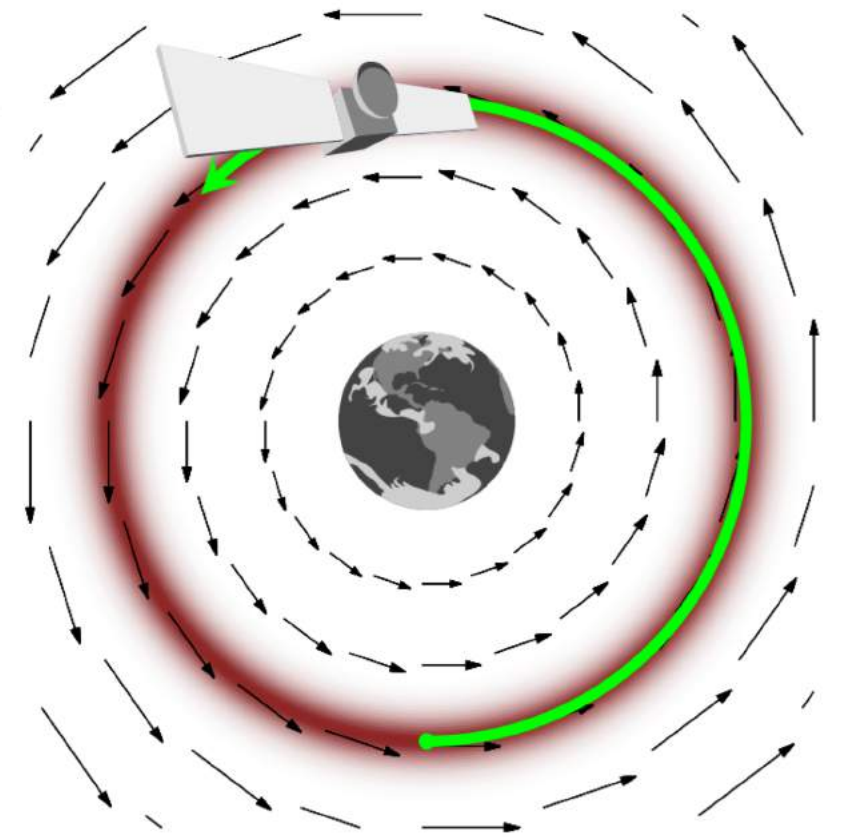
Typical set



Bad



Bad



Good



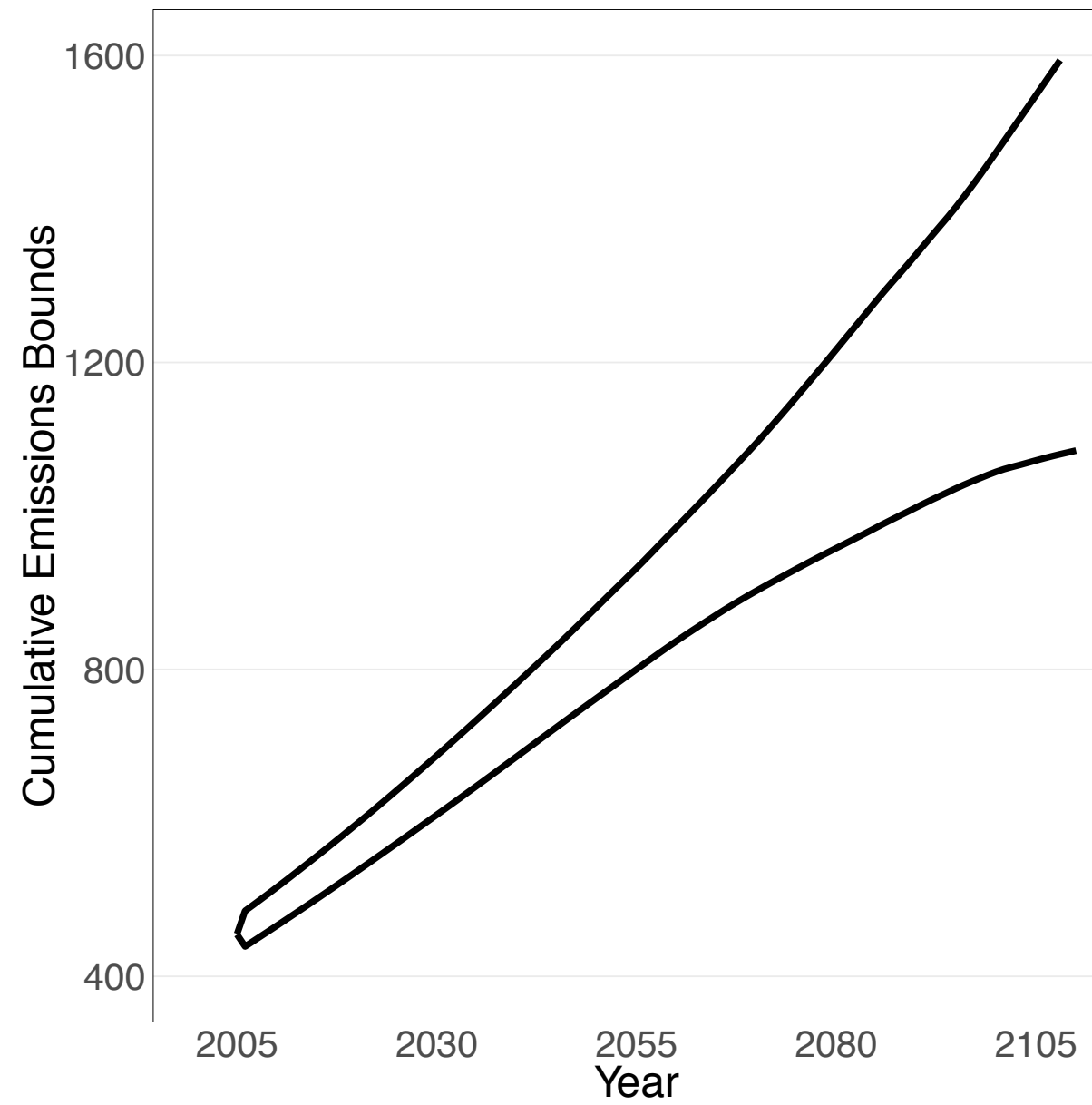
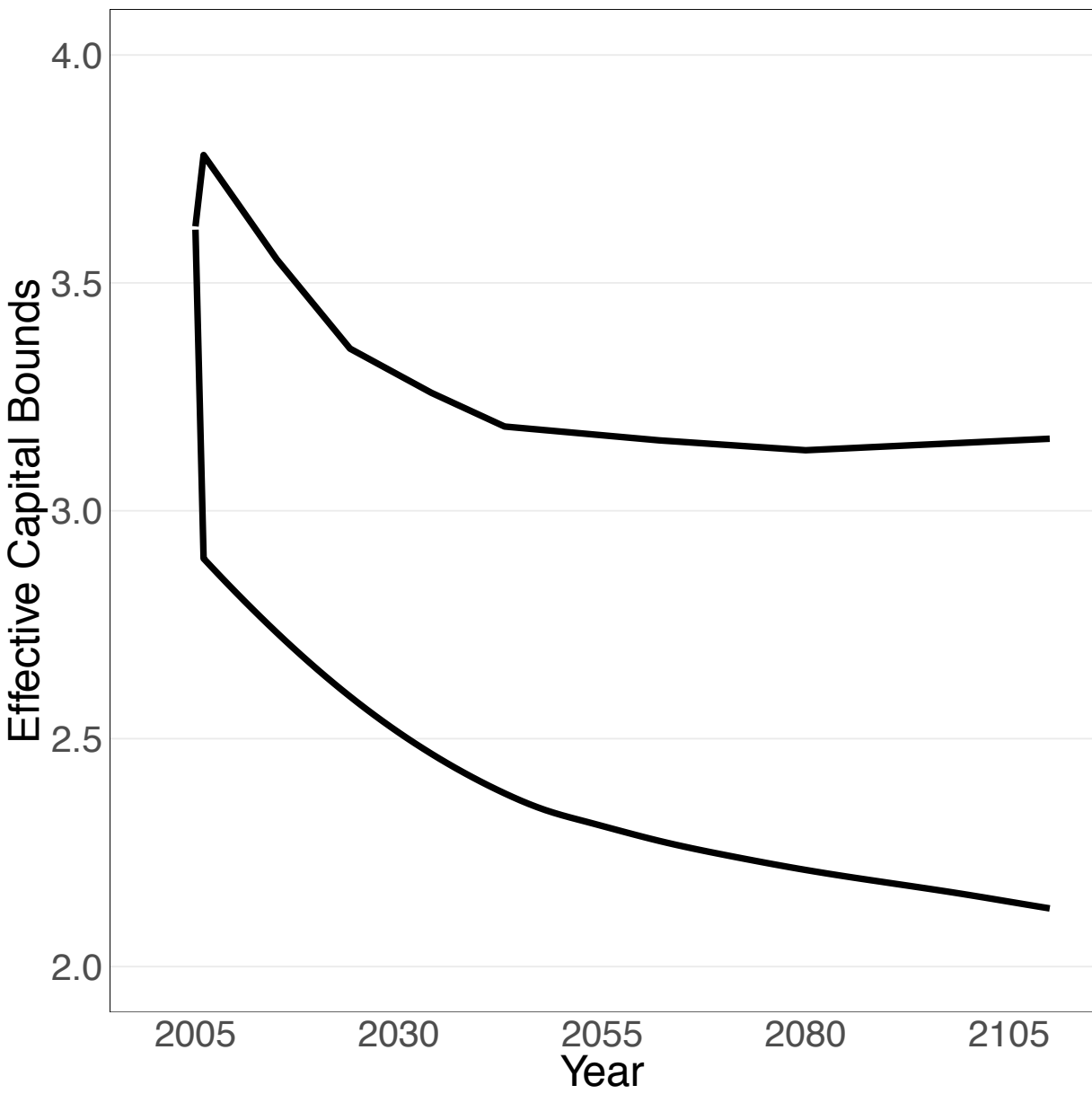
**Step 5: Solve the model (twice)**

**Dynamic  
Programming**

# The final step(s)

- 1) Do standard value function iteration
- 2) Simulate a bunch of potential state paths
  - The envelope of these paths will be time dependent
- 3) Generate a time-dependent / adaptive grid based on the sims
- 4) Repeat everything once

# Adaptive grids



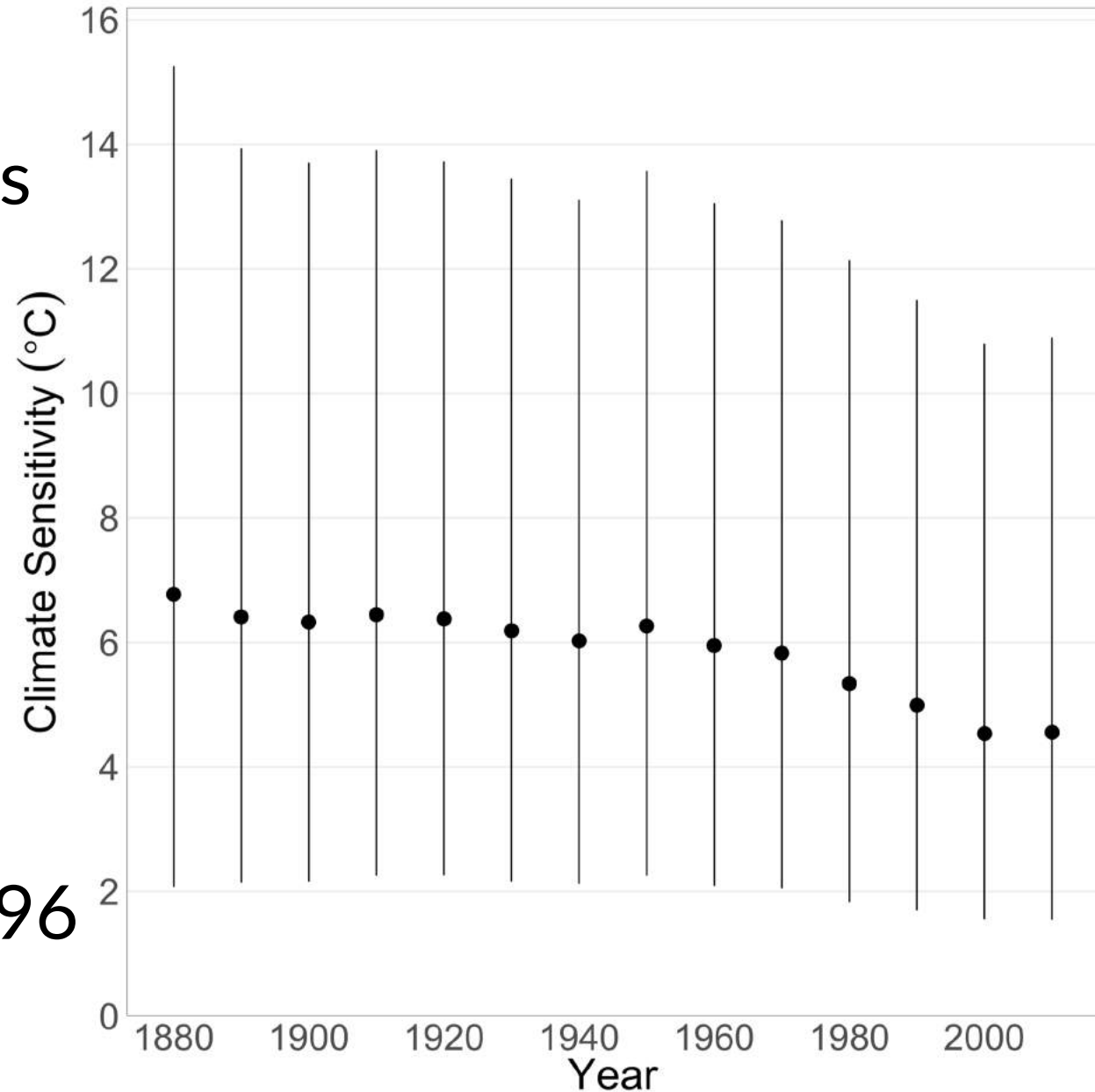
Finally,  
the results

# Backcast the learning model to validate

Conditional on the instrumental record, what belief trajectory gets us our current prior?

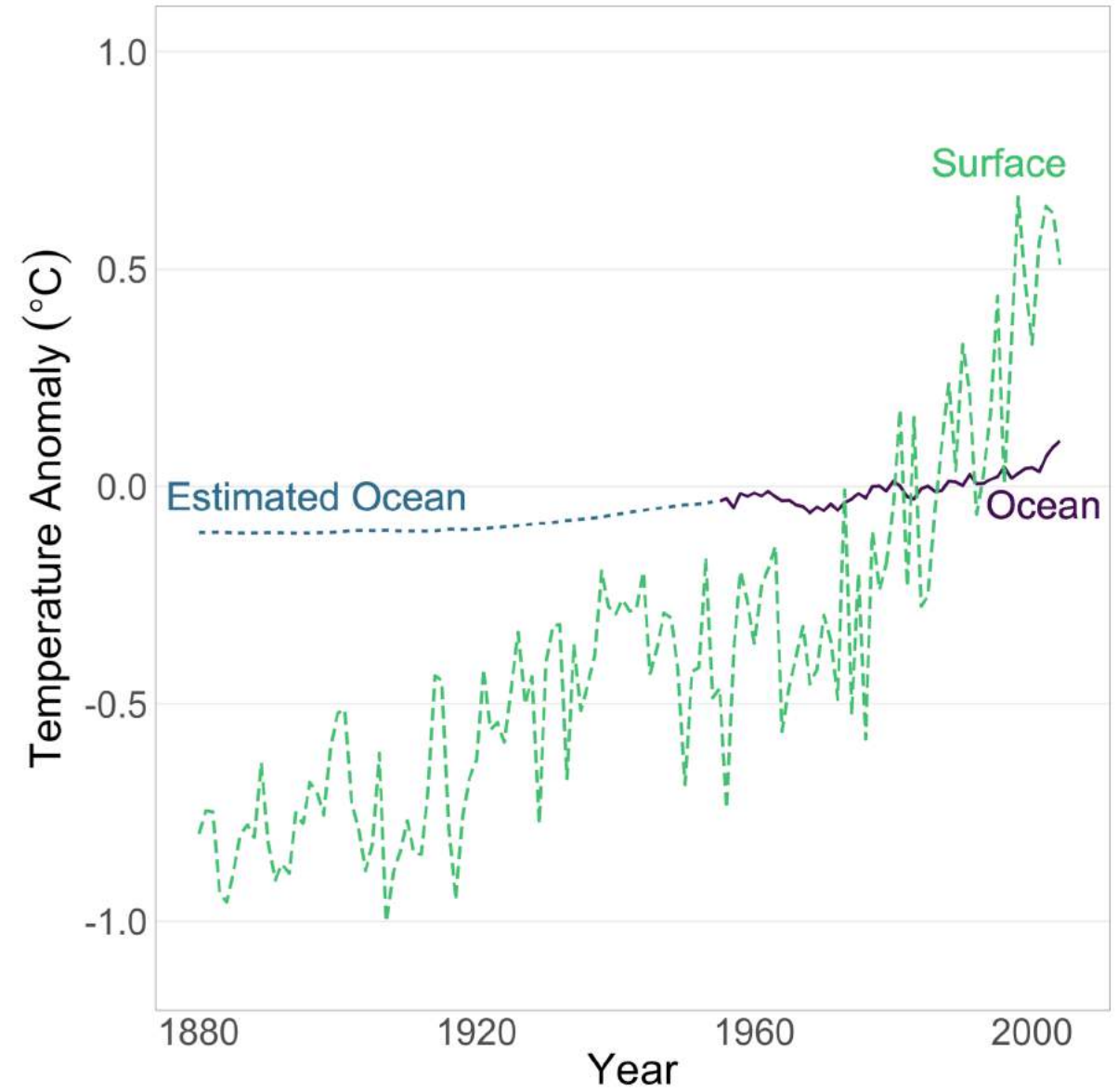
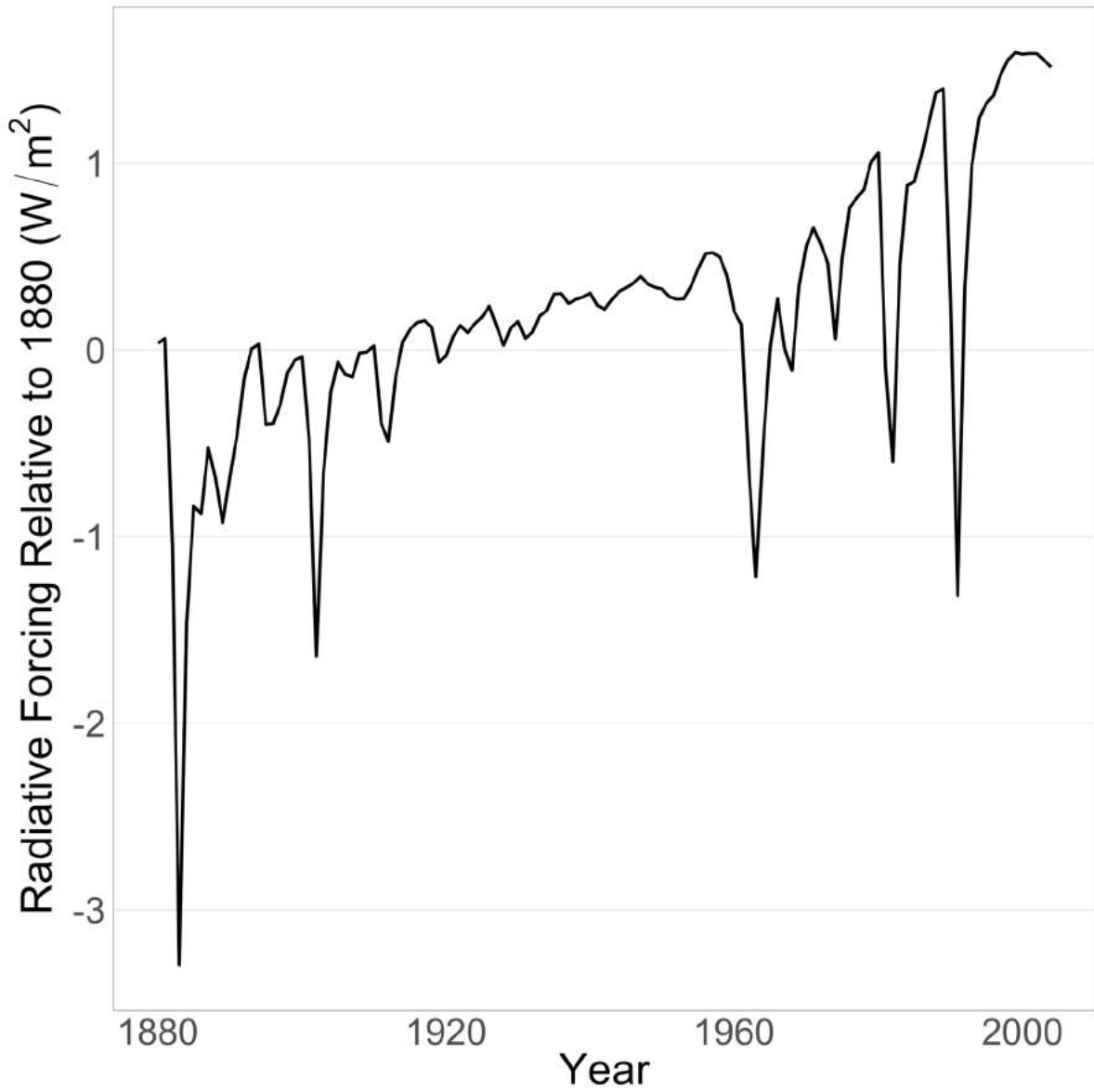
Consistent with historical beliefs over the 140 years

Even going back to the first climate sensitivity estimate in 1896





# The data and data estimates



# Advancing climate science increases the informativeness of the climate signal

We model improvements in modeling/monitoring as increasing the strength of the climate signal in the data

This is nearly identical to the theoretical information literature

- Radner and Stiglitz (1984); Chade and Schlee (2002); Moscarini and Smith (2002); Keppo et al. (2009)

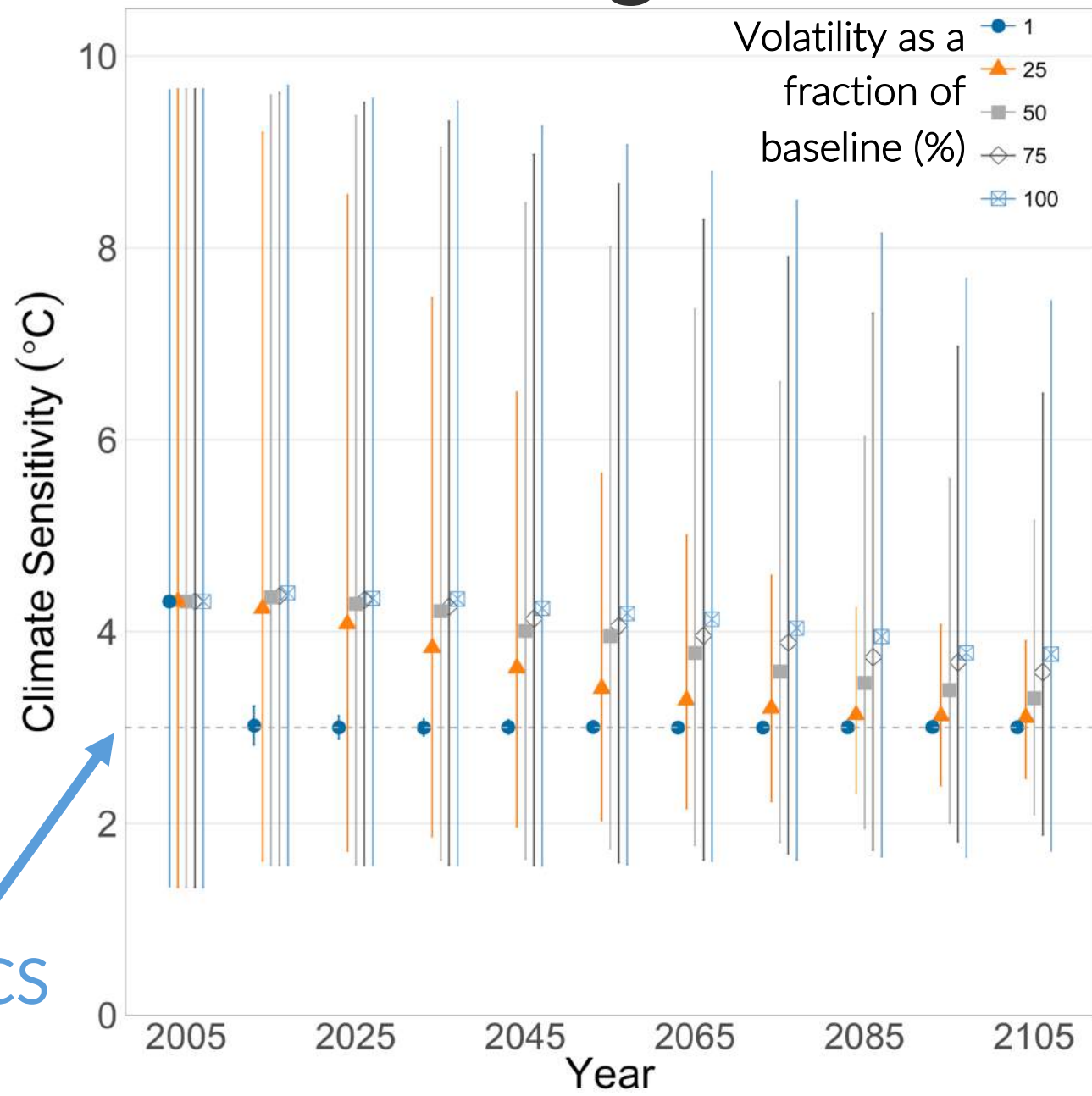
In practice, we reduce the volatility (unexplained variation) in temperature

# Reducing volatility accelerates learning

Mean and 90% CI of beliefs about CS under different temperature volatilities

Volatility reductions accelerate learning, allow for better policymaking

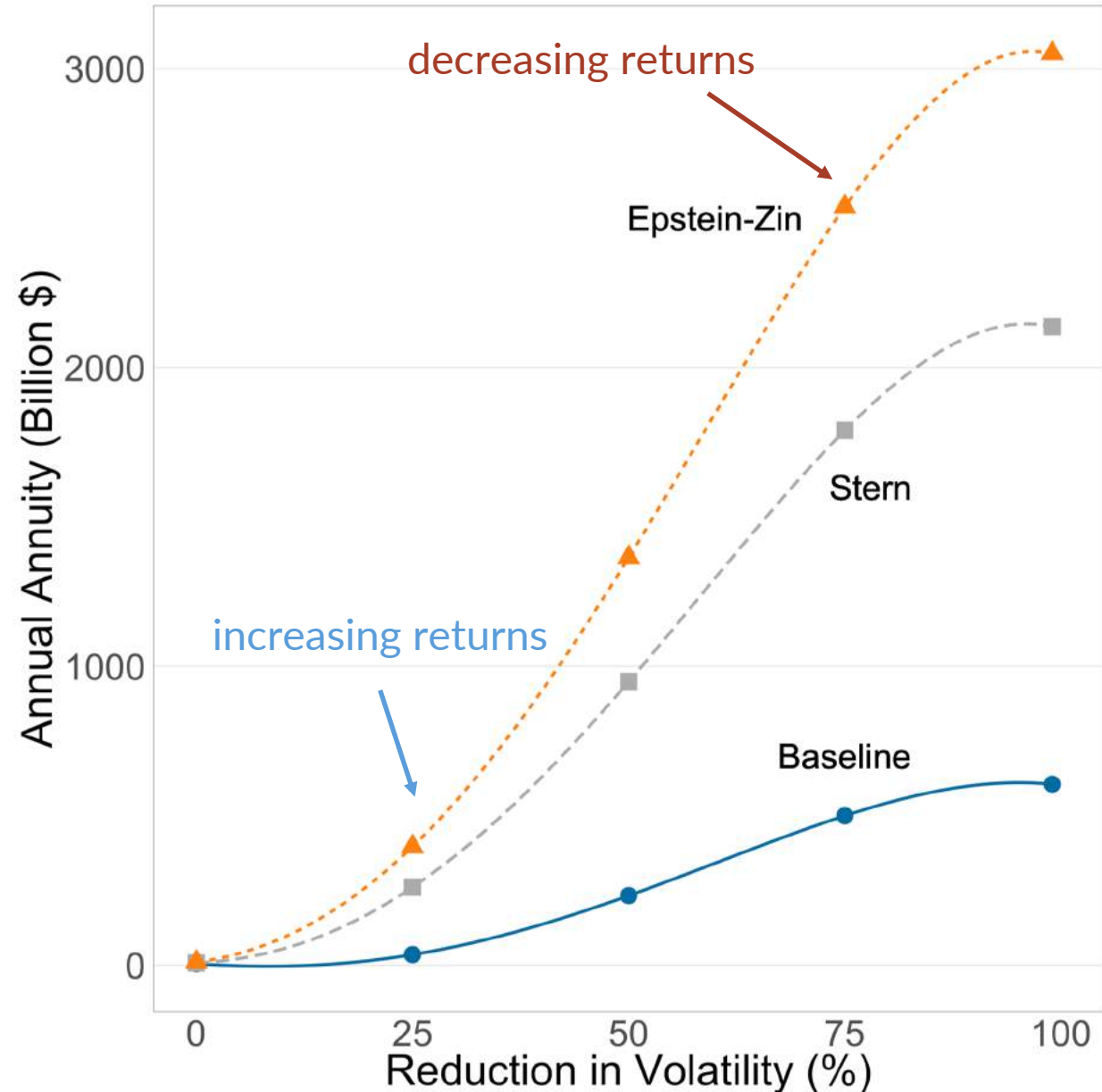
True CS



# Accelerating learning has significant value

Annuity from reducing unexplained variation in temperature

Non-concavities (upward sloping demand) are consistent with theoretical predictions



# We estimate the gains from accelerating science

Science policy is highly valuable

Optimal funding is zero or large,  
need MC estimates for optimal funding levels

Structural and numerical models often use  
similar methods to approximate functions and distributions

These methods make high dimensional problems feasible/faster

# Learn high dimensional optimization/approximation basics

Basis functions?

Gradient descent?

Sparse grids?

MCMC/HMC?

Julia?

## Step 4: Construct the grid and basis matrix

```
# Chebyshev polynomial function
function cheb_polys(x, n)
    if n == 0
        return 1 # T_0(x) = 1
    elseif n == 1
        return x # T_1(x) = x
    else
        cheb_recursion(x, n) =
            2x.*cheb_polys.(x, n - 1) .- cheb_polys.(x, n - 2)
        return cheb_recursion(x, n) # T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)
    end
end;
```

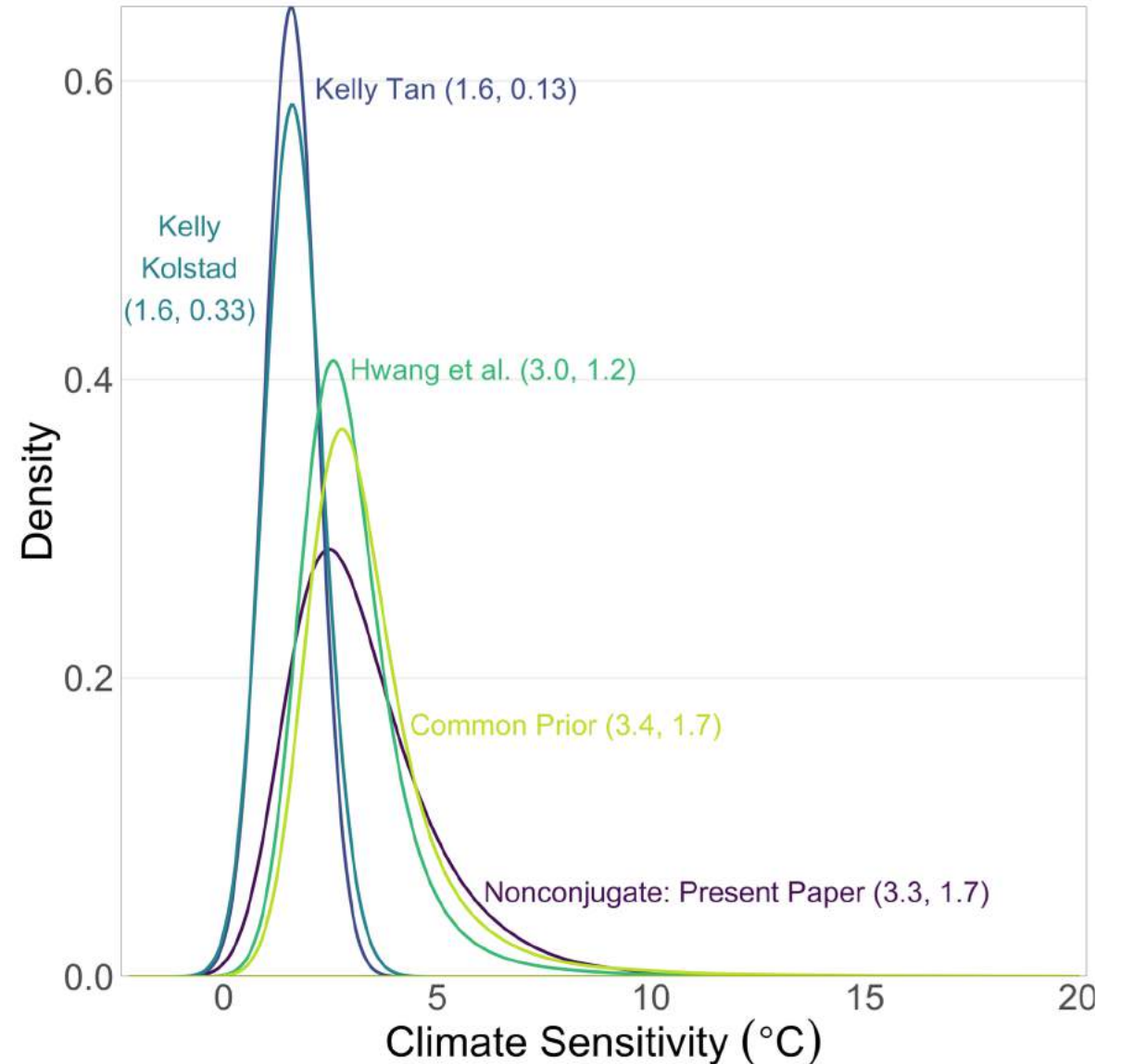


<https://github.com/AEM7130>



# Standard conjugate approach says we can now rule out disastrous climate sensitivity

Beliefs since the Charney report under four different IAM learning models



# Alternative nonconjugate learning models are biased in *a priori* nonobvious ways

